Lesson 23

ROTATE FIGURES USING COORDINATES

NY-8.G.3

WORDS TO KNOW

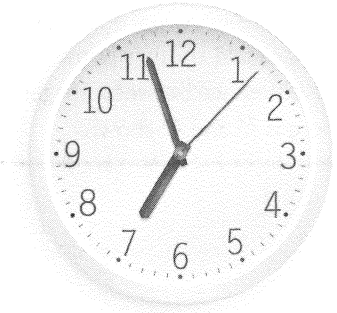
rotation

angle of rotation

center of rotation

INTRODUCTION

Real-World Connection



The hands on this clock are moving clockwise constantly. The second hand rotates 360° clockwise in one minute. Notice that the second hand is at seven seconds. Where will the second hand be after a 90° clockwise rotation? Let's practice the skills in the **Guided Instruction** and **Independent Practice** and find the location of the second hand at the end of the lesson!

What I Am Going to Learn

- How to rotate figures in the coordinate plane
- How the coordinates change when a figure is rotated

What I May Already Know

- I know how to plot points in the coordinate plane.
- I know how to draw a polygon in the coordinate plane, given coordinates of the vertices.
- I know how to find the lengths of sides in these polygons using the coordinates.

Vocabulary in Action

A **rotation** is a type of transformation.

- A rotation moves a point through an angle, called the **angle of rotation**. Angles of rotation are counterclockwise, unless otherwise stated.
- The point rotates around a point called the **center of rotation**. This point is often the origin.

When a figure rotates, its position changes, but not its size or shape.

- When a point is rotated about the origin, the angle of rotation determines the coordinates of the image.
- A 90° rotation moves a point (x, y) to $(-y, x)$.
- A 180° rotation moves a point (x, y) to $(-x, -y)$.

TIPS AND HINTS

If more than one image is shown, the vertices add another prime to tell the difference between images. So, there is A' and A'' shown.

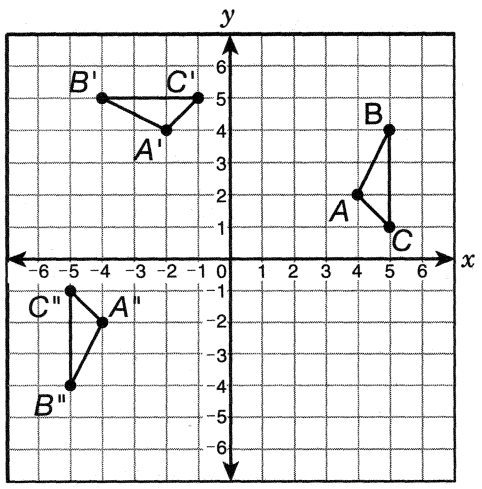
EXAMPLE

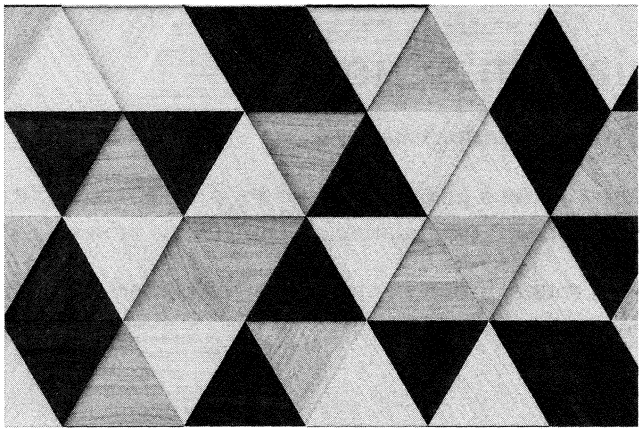
Triangle ABC is rotated about the origin 90° and 180°.

Show the coordinates of points B and C after each rotation.

90°: $B(5, 4) \rightarrow B'(-4, 5)$; $C(5, 1) \rightarrow C'(-1, 5)$

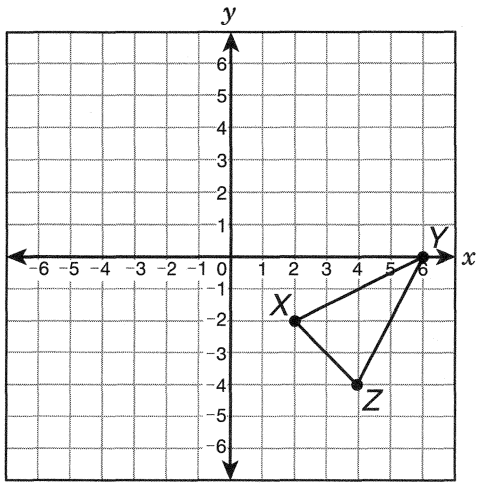
180°: $B(5, 4) \rightarrow B''(-5, -4)$; $C(5, 1) \rightarrow C''(-5, -1)$





GUIDED INSTRUCTION

1. Plot the image of triangle XYZ after a 180° rotation.



Step One Identify the coordinates of the triangle.

$X(2, -2)$

$Y(6, 0)$

$Z(4, -4)$

Step Two Write the coordinates for the image of each point.

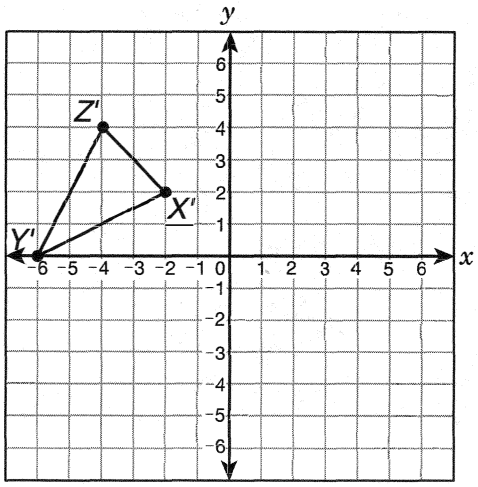
For a 180° rotation, $(x, y) \rightarrow (-x, -y)$

$X(2, -2) \rightarrow X'(-2, 2)$

$Y(6, 0) \rightarrow Y'(-6, 0)$

$Z(4, -4) \rightarrow Z'(-4, 4)$

Step Three Plot the new points and draw the image of the triangle.



THINK ABOUT IT

If a coordinate is zero, it will be on an axis. Its image will also be on the axis. Zero does not have an opposite.

TURN AND TALK

What rotation would be the same as a 90° counterclockwise rotation?

2. What would the coordinates of the image of triangle XYZ be if the figure were rotated 90°?

Step One Identify the coordinates of the triangle.

$X(2, -2)$

$Y(6, 0)$

$Z(4, -4)$

Step Two Write the coordinates for the image of each point.

For a 90° rotation, $(x, y) \rightarrow (-y, x)$

$X(2, -2) \rightarrow X'(2, 2)$

$Y(6, 0) \rightarrow Y'(0, 6)$

$Z(4, -4) \rightarrow Z'(4, 4)$

TIPS AND HINTS

Rotating a point could change the signs of the coordinates, the order of the coordinates, or both. Carefully examine the ordered pairs before making your decision.

3. Which one of the following rotations produces an image at $(-4, -8)$?

Ⓐ Rotate the point (4, 8) 180°.

Ⓑ Rotate the point (4, −8) clockwise 90°.

Ⓒ Rotate the point (8, −4) clockwise 90°.

Ⓓ Rotate the point (8, 4) 180°.

Learning Together

Working with a partner, find images of things that rotate, such as a merry-go-round, a frisbee, and a spinning arrow for a game. Tape the images on a piece of paper. In a note next to each image, indicate whether, if placed on a coordinate grid, it could rotate about the origin or would likely rotate more randomly. Also, for each item, mark a beginning point and label it 0°. Then use slash marks and arrows to mark and label rotation points at 90° and 180°.

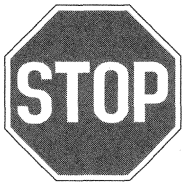
How Am I Doing?

What questions do you have?

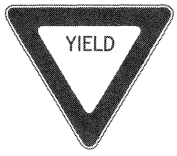
In the space below, draw a figure and a rotation of the same figure.

What is an example of something that rotates about a point and changes position?

Circle the sign that shows how you are doing with the skill.



I am stuck.



I almost have it.

I understand the skill.

INDEPENDENT PRACTICE 1

1 Triangle $X'Y'Z'$ is located at $X'(-3, 7)$, $Y'(2, 5)$, and $Z'(-1, -1)$. The pre-image was rotated 90° counterclockwise about the origin. Which is a coordinate of the pre-image?

A (7, 3)

B (1, 1)

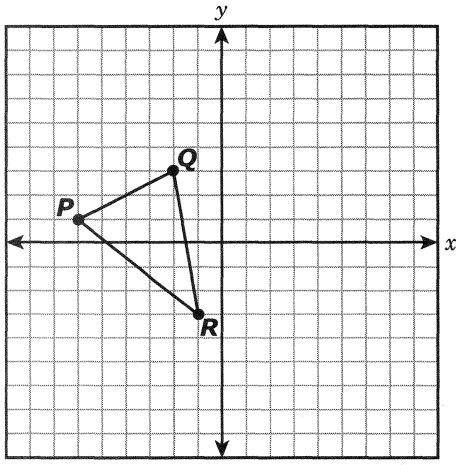
C (−7, −3)

D (1, −1)

TIPS AND HINTS

Write out coordinates of the pre-image before looking at the choices. Then, look at the answer choices and compare.

2 Triangle *PQR* is rotated 90° counterclockwise about the origin.



TIPS AND HINTS

Remember that algebraic rules for rotation equations include both the pre-image and the image.

Which of the following shows the algebraic rule applied to one of the vertices?

A $(3, -1) \rightarrow (-1, -3)$

B $(-6, 1) \rightarrow (-6 + 5, -6 + 0)$

C $(-2, 3) \rightarrow (3, -(-2))$

D $(-1, -3) \rightarrow (-(-3), -1)$

3 Point X $(-9, 3)$ can be rotated to produce point X' $(3, 9)$. What is the measure of the angle of rotation?

A 90°

B 180°

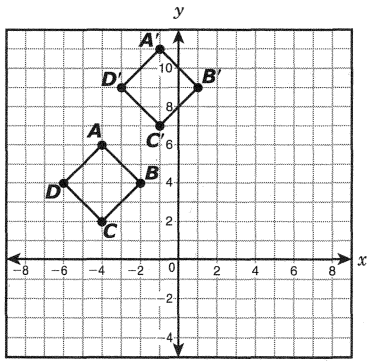
C $-9 - 3 = x$

D $3 - 9 = x$

THINK ABOUT IT

Are angles measured in degrees or with equations?

4 A transformation is shown below on the coordinate grid.



TIPS AND HINTS

Rules for rotating about the origin:

90°: $(x, y) \rightarrow (-y, x)$

180°: $(x, y) \rightarrow (-x, -y)$

Could it be a rotation about the origin?

Answer _____

Explain your answer.

INDEPENDENT PRACTICE 2

1 A Ferris wheel is drawn on a coordinate plane so that the coordinates of the first car are (0, 8). What are the coordinates of the first car after the Ferris wheel rotates 180° about the origin?

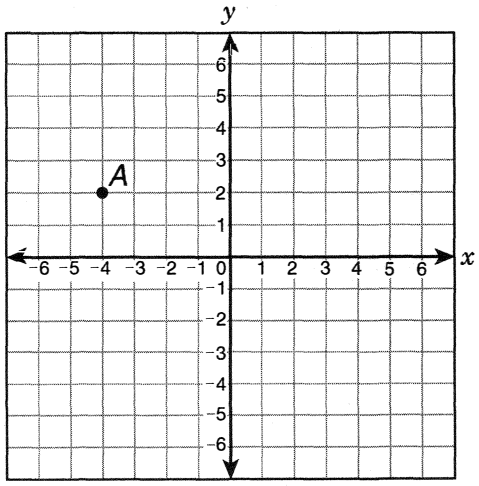
A (8, 0)

B (0, −8)

C (−8, 0)

D (0, 0)

2 On the grid below, point *A* is rotated clockwise 90° and 180° about the origin.



What are the coordinates of the image for each angle of rotation?

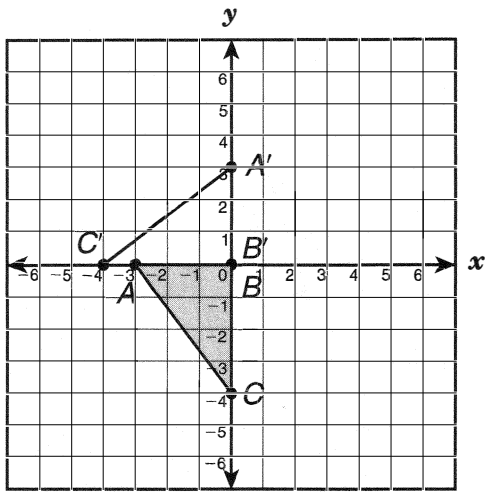
A 90°: (2, 4); 180°: (4, −2)

B 90°: (−2, −4); 180°: (4, 2)

C 90°: (−2, 4); 180°: (−4, 2)

D 90°: (2, −4); 180°: (−4, −2)

3 The grid below shows a triangle and its image after a transformation.



Which statement most accurately describes the figures on the coordinate grid?

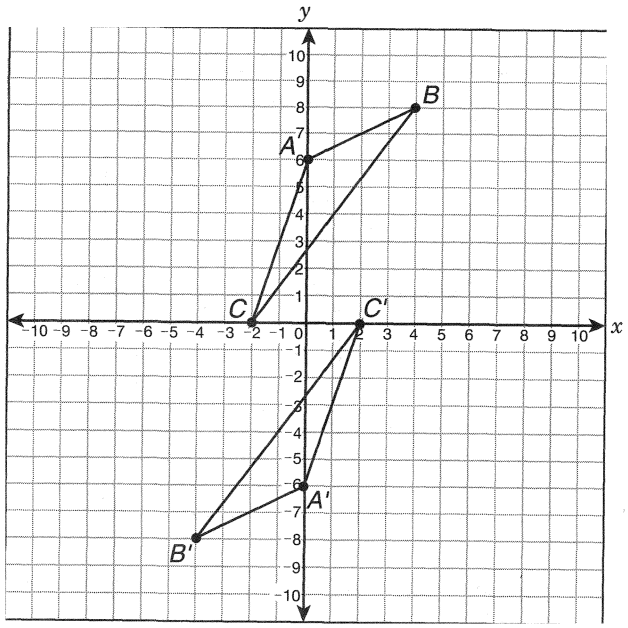
A It does not represent a rotation, because $(x, y) \rightarrow (-y, x)$.

B It does not represent a rotation, because $(x, y) \rightarrow (y, -x)$.

C It represents a rotation, because $(x, y) \rightarrow (-y, x)$.

D It represents a rotation, because $(x, y) \rightarrow (y, -x)$.

4 The graph below shows the pre-image ABC and the image $A'B'C'$.



What is the measure of the angle of rotation?

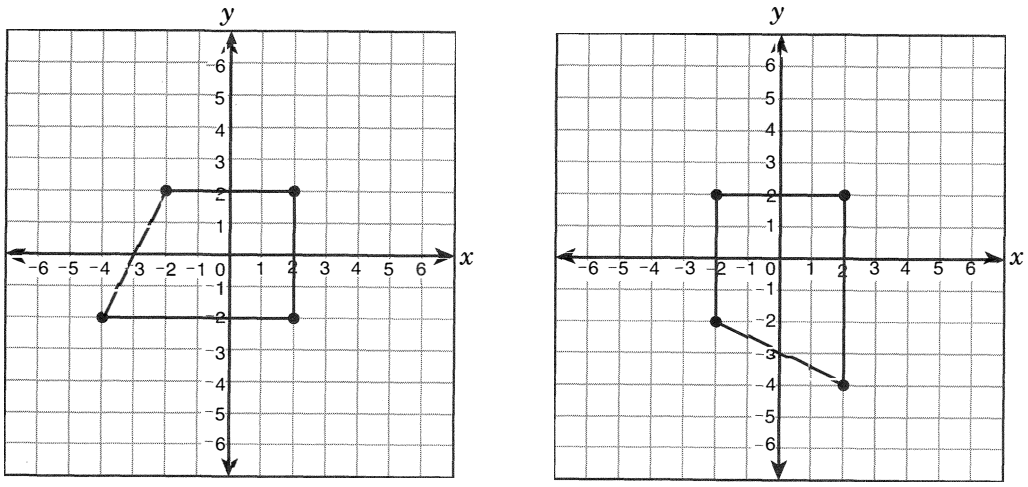
A 90°

B −90°

C 180°

D −180°

5 The grid to the left below shows a coordinate plane with a quadrilateral. The grid to the right below shows the quadrilateral after a rotation.



Which choice correctly describes the translation?

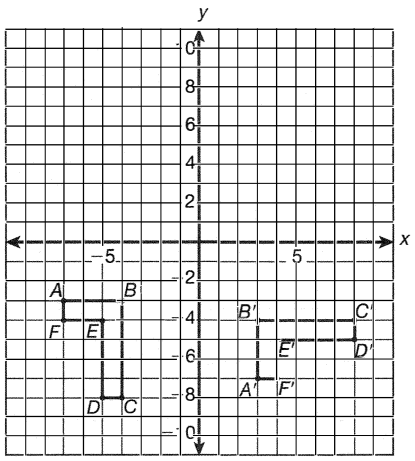
A 90° clockwise

B 90° counterclockwise

C 180° clockwise

D 180° counterclockwise

6 Magda drew a figure on a coordinate plane and transformed it as shown below.



Which of the following **best** describes Magda's transformation?

A She rotated the figure 90° counterclockwise about the origin.

B She reflected the figure over the *y*-axis.

C She translated the figure seven units right and one unit down.

D She rotated the figure 180° about the origin.

7 A figure has a vertex at (a, b), where a is a positive integer and b is a negative integer. What is the algebraic rotation rule that takes the figure to Quadrant I?

A $(x, y) \rightarrow (y, -x)$

B $(x, y) \rightarrow (-x, -y)$

C $(x, y) \rightarrow (-y, x)$

D $(x, y) \rightarrow (x, y)$

8 Point R is at $(-9, -2)$ on a circle with center (0, 0). After a rotation of 180° about the center, what are the coordinates of point R'?

Answer _____

Explain your answer.

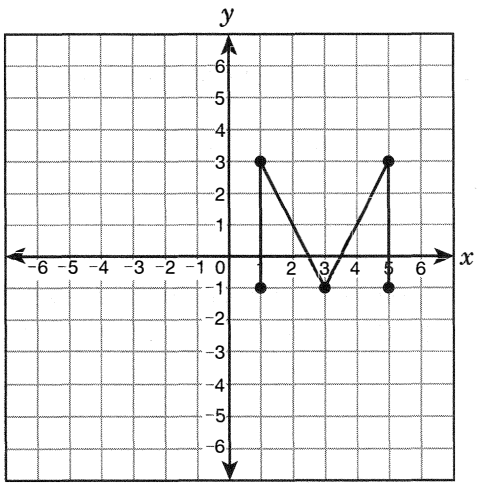
_____

_____

_____

_____

9 The capital letter M can be turned in to the capital letter W using a rotation about the origin. State the angle of rotation that would change the M shown below into a W and give the coordinates of the image points.



Answer _____, _____, _____, _____, _____

Explain your answer.

_____

_____

_____

_____

EXIT TICKET

NY-8.G.3

Now that you have mastered rotating a figure, let's solve the problem in the **Real-World Connection.**

The hands on this clock are moving clockwise constantly. The second hand rotates 360° in one minute. Notice that the second hand is at 7 seconds. Where will the second hand be after a 90° clockwise rotation? Explain how you arrived at your answer.